WTC 7: A short computation

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Introduction

I provide a short computation, focused on World Trade Center building 7. Based on very favorable assumptions for achieving a fast fall, including ignoring resistance due to intact steel columns, I could only get the building to fall in about 8.3 seconds, whereas the observed roof-fall time is approximately 6.5 seconds. The problem is the large number of floors and conservation of momentum in a collision. Some of the "official" explanations about progressive collapse are evocative but they do not explain the difficulty in the rapid fall of the building along with what is evidently taking place when the video of the falling building is observed.

A short computation

Building 7 was 576 feet (176 m) tall. The speed of a ball bearing falling from the top of this building to the ground is therefore the solution to the equation

$$\frac{1}{2}v^2 = gh = 32\times 576$$

which yields $v = 192$. Thus the time to fall to the ground would be

$$\frac{576}{\left(\frac{192}{2}\right)} = 6 \text{ seconds.}$$

It was observed that the building collapsed in just 6.5 seconds.[1] Could this possibly happen as a result of pancaking floors collapsing from the top down? We show here that if the collisions are inelastic, such a scenario is impossible.

Assume there is not support for any floor when it is hit by the collapsing floors from above. Thus it is like the floor is just floating in the air when it is hit but it is stationary.

To make things general, let h denote the spacing between floors and let there be $n > 2$ of these floors.

Let v_k be the velocity of the conglomeration of k of the floors just before it hits the (k+1)st floor and let $\widehat{v_{k+1}}$ denote the velocity of the larger conglomeration of floors immediately after the collision. Then by conservation of momentum

$$kMv_k = (k+1)M\widehat{v_{k+1}}$$

so

$$\widehat{v_{k+1}} = \tfrac{k}{k+1}v_k.$$

The bottom side of this enlarged conglomeration falls another *h* feet before hitting the next floor. By conservation of energy,

$$\frac{1}{2}(k+1)M\left(\frac{k}{k+1}v_k\right)^2+(k+1)Mgh=\frac{1}{2}(k+1)Mv_{k+1}^2$$

where *g* is the acceleration of gravity. Thus, solving for v_{k+1} yields

$$v_{k+1}=\sqrt{\left(\frac{k}{k+1}v_k\right)^2+2gh} \qquad (1)$$

The average speed of the falling $k+1$ floors during the time between these two collisions is

$$\frac{1}{2}\left(\overbrace{\sqrt{\frac{k^2}{(k+1)^2}v_k^2+2gh}}^{v_{k+1}}+\overbrace{\frac{k}{k+1}v_k}^{\widehat{v_{k+1}}}\right).$$

Iterating 1 using $v_0 = 0$ yields

$$v_1=\sqrt{2gh},v_2=\sqrt{\left(\left(\tfrac{1}{2}\right)^2+1\right)2gh},v_3=\sqrt{\left(\left(\tfrac{1}{3}\right)^2+\left(\tfrac{2}{3}\right)^2+1\right)2gh}$$

$$v_4=\sqrt{\left(\tfrac{3}{4}\right)^2\left(\left(\left(\tfrac{1}{3}\right)^2+\left(\tfrac{2}{3}\right)^2+1\right)2gh\right)+2gh}$$

$$=\sqrt{\left(\left(\tfrac{1}{4}\right)^2+\left(\tfrac{2}{4}\right)^2+\left(\tfrac{3}{4}\right)^2+1\right)2gh}$$

This leads to the following formula for v_k, the velocity of the falling conglomeration of floors just before striking the $(k+1)^{st}$ floor.

$$v_k=\sqrt{\left(1+\frac{1}{k^2}\sum_{j=1}^{k-1}j^2\right)2gh}$$

The total time for falling, neglecting any time for the collisions themselves would then be

$$\frac{h}{\left(\frac{\sqrt{2gh}}{2}\right)}+\frac{h}{\frac{1}{2}\left(\sqrt{\left(\left(\frac{1}{2}\right)^2+1\right)+2gh}+\frac{1}{2}\sqrt{2gh}\right)}$$

$$+\sum_{k=3}^{n}\frac{h}{\left(\frac{1}{2}\sqrt{\left(1+\frac{1}{k^2}\sum_{j=1}^{k-1}j^2\right)2gh}+\frac{k-1}{k}\sqrt{\left(1+\frac{1}{(k-1)^2}\sum_{j=1}^{k-2}j^2\right)2gh}\right)}$$

Some simplifying assumptions: The building was 576 feet tall and there were 47 floors of mass M which were equally spaced. Thus the distance between floors is $\frac{576}{47} = 12.2553191$ or about 12 feet. Assuming the thickness of each floor is 33 inches, this would yield a value for h of

$$h = \left(\frac{576}{47} - \frac{33}{12}\right) = 9.5 \text{ feet.}$$

Inserting this into the above formula using $g = 32$ ft. per sec^2,

$$2gh = 64 \times 9.5 = 608.$$

Thus the above series reduces to

$$\frac{9.5}{\left(\frac{\sqrt{608}}{2}\right)} + \frac{9.5}{\frac{1}{2}\left(\sqrt{\left(\left(\frac{1}{2}\right)^2 + 1\right)608} + \frac{1}{2}\sqrt{608}\right)}$$

$$+\sum_{k=3}^{47} \frac{9.5}{\frac{1}{2}\left(\sqrt{\left(1 + \frac{1}{k^2}\sum_{j=1}^{k-1} j^2\right)608} + \frac{k-1}{k}\sqrt{\left(1 + \frac{1}{(k-1)^2}\sum_{j=1}^{k-2} j^2\right)608}\right)}.$$

Letting the computer do the computation this yields a total of 8.335 seconds. (For floors 2 feet thick, the fall time becomes 8.55 seconds.)

Now consider what the above computation describes. It yields a stack of 47 floors, each of depth 33 inches resulting in a pile which is $47 \times \frac{33}{12} = 129.25$ feet high. The collapse of the building was far more complete than this but even this very incomplete collapse would take longer to achieve than the 6.5 seconds observed for the fall of the building's roof. It follows there is something wrong with the assumptions. Perhaps the collisions are not inelastic; but they were clearly not elastic in view of the observed pulverization.

The other possibility is that the building fell in such a way that the falling floors encountered very little resistance until they reached the bottom. This possibility seems more likely, especially when the videos of the building are observed.[2]

Footnotes

1. Observing the southwest corner of the roof as it begins its steady fall. S.E. Jones, "Why Indeed Did the WTC Buildings Collapse?" in 9/11 And The American Empire: Intellectuals Speak Out, David Ray Griffin and Peter Dale Scott, eds., 2006.

2. Any further analysis of the collapse of WTC 7 should include all floors (not just "floors 8 to 46") and conservation of momentum considerations (see http://wtc.nist.gov/solicitations/wtc_awardQ0186.htm).